Title: Curve of Best Fit -- The Swinging Pendulum

Brief Overview:

The students will use experimental data and the regression feature of the TI-83 graphing calculator to develop a model that will predict the time period of a pendulum swing or the length of the pendulum. After a review of linear regression on the TI-83, the students will input the data from their conducted experiments into the TI-83, construct a scatter plot, perform quadratic, power, and exponential regressions, compare and contrast curves of best fit, and answer question based on the selected model.

NCTM 2000 Principles for School Mathematics:

- **Equity:** *Excellence in mathematics education requires equity - high expectations and strong support for all students.*

- **Curriculum:** *A curriculum is more than a collection of activities: it must be coherent, focused on important mathematics, and well articulated across the grades.*

- **Teaching:** *Effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well.*

- **Learning:** *Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge.*

- **Assessment:** *Assessment should support the learning of important mathematics and furnish useful information to both teachers and students.*

- **Technology:** *Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning.*

Links to NCTM 2000 Standards:

- **Content Standards**

  Algebra
  Students will understand patterns, relations, and functions. They will represent and analyze mathematical situations and structures using algebraic symbols. Mathematical models will be developed to represent and understand quantitative relationships. Students will be able to draw reasonable conclusions about the situation being modeled.

  Geometry
  Students will use Cartesian coordinates to represent the data and the model.

Measurement
Students will understand measurable attributes of objects and the units, systems, and processes of measurement. They will apply appropriate techniques, tools, and formulas to determine measurements.

Data Analysis and Probability
Students will understand scatter plots and use them to display data. They will determine regression equations that model the data using technological tools. The students also will develop and evaluate inferences and predictions that are based on data.

- **Process Standards**

Mathematics as Problem Solving, Reasoning and Proof, Communication, Connections, and Representation
These five process standards are threads that integrate throughout the unit. They emphasize the need to help students develop the processes that are the major means for doing mathematics, thinking about mathematics, understanding mathematics, and communicating mathematics.

Students will use authentic and experimental data to analyze and predict the length of a pendulum. They must make appropriate conclusions, and use correct mathematical language to justify and explain their conclusions. Students also will represent their data in tabular, graphical, and functional forms.

Links to Maryland High School Mathematics Core Learning Units:

Functions and Algebra

- **1.1.1**
  Students will recognize, describe, and extend patterns and functional relationships that are expressed numerically, algebraically, and geometrically.

- **1.1.2**
  Students will represent patterns and functional relationships in a table, as a graph, and/or by mathematical expression.

- **1.2.1**
  Students will determine the equation for a line, solve linear equations, and describe the solutions using numbers, symbols, and graphs.

- **1.2.4**
  Students will describe how the graphical model of a non-linear function represents a given problem and will estimate the solution.

<u>Data Analysis and Probability</u>

- **3.1.1**
  Students will design and/or conduct an investigation that uses statistical methods to analyze data and communicate results.

- **3.2.2**
  Students will make predictions by finding and using a line of best fit and by using a given curve of best fit.

- **3.2.3**
  Students will communicate the use and misuse of statistics.

Links to National Science Education Standards:

- **Science as Inquiry**
  Students will learn skills, such as observation, inference, and experimentation.

Grade/Level:

Grades 9 – 12, Algebra II

Duration/Length:

Three day lesson and one day assessment - 45 minute periods.

Prerequisite Knowledge:

Students should have working knowledge of the following skills:

- Constructing scatter plots
- Writing an equation of a line
- Graphing linear and quadratic equations on a coordinate plane
- Understanding the units and meaning of slope and y-intercept
- Determining domain and range values for linear and quadratic functions
- A priori knowledge of the TI-83 graphing calculator, including graphing, scatter plots, and linear regressions

Student Outcomes:

Students will:

- construct scatter plots on the TI-83 using authentic data.
- determine and interpret regression equations using authentic data.
- use regression equations to model and predict pendulum lengths or swing period.
- use the list, regression, table, and plot functions of the graphing calculator.

Materials/Resources/Printed Materials:

- TI-83 or TI-83 Plus graphing calculator (classroom set)
- TI-83 overhead projector
- String (6 feet per group)
- Large / heavy washer (one per group)
- Stop watch (one per group)
- Protractor (one per group)
- Yard stick (one per group)
- Data Analysis instructions for the graphing calculator
- Worksheets, Assessment

Development/Procedures:

Day 1
Students will review linear regression using the TI-83 graphing calculator. The teacher-directed lesson will require students to input provided data, construct a scatter plot, calculate the line of best fit, note the correlation coefficient, draw the residual plot, determine the sum of the squares of the residuals, answer questions, and make predictions. (See Worksheet 1.)

Day 2
Students will conduct an experiment collecting data on length of the pendulum versus the period of the swing. Each group of students will measure the period for four different pendulum lengths and share their information with the other groups in order to generate a large list of data. The students will then input the data into the TI-83 graphing calculator and construct a scatter plot. (See Worksheet 2.)

Day 3
Since the scatter plot indicates a non-linear relationship, the students will perform power, exponential, and quadratic regressions on the TI-83 graphing calculator. They will conduct analyses to determine the best model to predict the length or the period of the pendulum. The analysis will require the students to compare and contrast the residual plot, the correlation coefficient, the sum of the square of residuals, and the graph of the data and line of best fit for each regression scenario. (Continues on Worksheet 2.)

Day 4
Students will be assessed on what they have learned with two brief constructed response problems and one extended constructed response problem. (See Assessment.)

Assessment:

The students will be assessed on their ability to use an equation for a line of best fit to calculate the period of the pendulum swing and a separate problem to determine the length of a pendulum. They will also be assessed on their ability to take a set of data and perform a non-linear regression using the features in the TI-83 graphing calculator.

Extension/Follow Up:

- Students can perform regression on sets of data found on the Internet.
  Example sites are:
  http://archives math.utk.edu/topics/statistics.html
  http://www.amstat.org/publications/jse/archive.htm

- Students can perform regression on sets of data found in various books
  Example textbook is Allan J. Rossman and J. Barr Von Oehsen, Workshop Statistics Discovery with Data and the Graphing Calculator, 1997 Springer-Verlag New York, Inc.

Authors:

Douglas Nelson
Dulaney High School
Baltimore County, MD

Russell Rushton
Walt Whitman High School
Montgomery County, MD

NAME ________________________________

DATE ____________

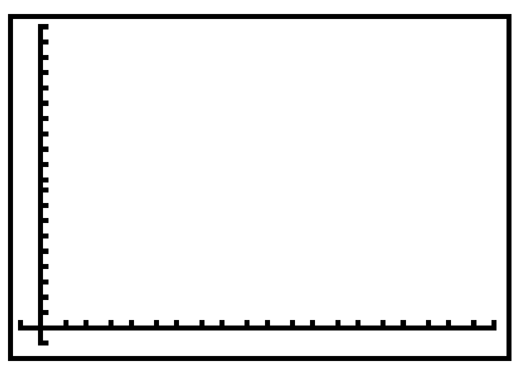
WORKSHEET #1: REVIEW OF LINEAR REGRESSION FROM ALGEBRA 1

The data below was obtained from the Florida Marine Patrol in Tallahassee, and it relates the number of deaths of manatees to the number (in thousands) of powerboat registrations from 1977-1994. (Source: Yates, Moore, and McCabe. The Practice of Statistics. W. H. Freeman and Company, 1999. p.165.)

Your task is to determine whether there is a linear relationship between the number of powerboat registrations and the number of manatees killed. Your task is to find the "line of best fit" for the data in the table below. Be able to explain why the data set is linear.

YEAR	Powerboat Registrations (in thousands)	Manatees Killed
1977	447	13
1978	460	21
1979	481	24
1980	498	16
1981	513	24
1982	512	20
1983	526	15
1984	559	34
1985	585	33
1986	614	33
1987	645	39
1988	675	43
1989	711	50
1990	719	47
1991	716	53
1992	716	38
1993	716	35
1994	735	49

1. Under Stat Edit, enter the registration and manatee data into lists L1 and L2, respectively, in your TI-83 graphing calculator, and sketch your scatter plot below.



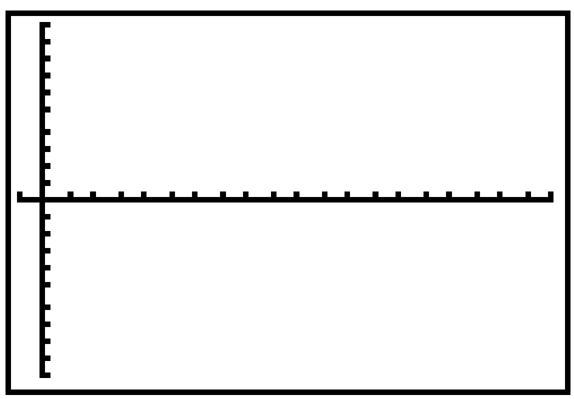
NAME ________________________________

DATE ___________

2. Determine the equation of the "best fit line" using linear regression. Use your table to determine the predicted number of manatee deaths if there were 900,000 powerboat registrations. (Remember that the table and your work have been based on *thousands* of powerboat registration.)

3. What was your correlation coefficient? What does this value tell you about your linear regression line?

4. Construct a residual plot (store RESID into L4 and do a scatter plot using L1 and L4). Sketch the graph below. Be sure to label and scale your axes.



5. Comment on the shape of your residual plot and how that relates to your model. Are your residuals random (i.e., not in a pattern) and small (relatively close to zero)? Based on this answer, does your linear regression line seem to fit the data well? Justify your answer.

6. Compute the sum of the squares of the residuals for your data. Explain the process you went through to accomplish this.

ANSWER KEY

NAME ______________________________

DATE ____________

WORKSHEET #1: REVIEW OF LINEAR REGRESSION FROM ALGEBRA 1

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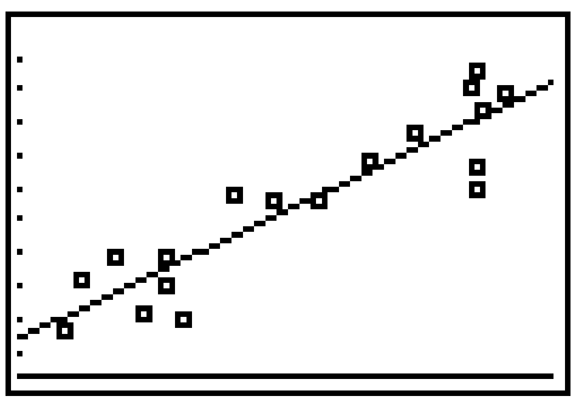


NAME ___________________________________

DATE ____________

2. Determine the equation of the "best fit line" using linear regression. Use your table to determine the predicted number of manatee deaths if there were 900,000 powerboat registrations. (Remember that the table and your work have been based on *thousands* of powerboat registration.)

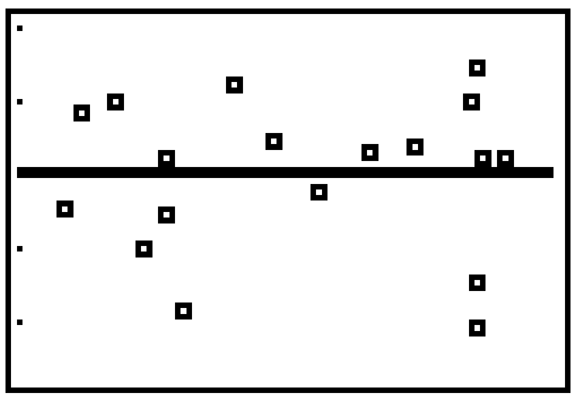
Linear Regression Line is Y = 0.1127X - 35.1786

For 900,000 registrations (or 900 in the table), the predicted deaths are 66 manatee deaths per year

3. What was your correlation coefficient? What does this value tell you about your linear regression line?

Correlation Coefficient r = .912657 (very good fit)

4. Construct a residual plot (store RESID into L4 and do a scatter plot using L1 and L4). Sketch the graph below. Be sure to label and scale your axes.



5. Comment on the shape of your residual plot and how that relates to your model. Are your residuals random (i.e., not in a pattern) and small (relatively close to zero)? Based on this answer, does your linear regression line seem to fit the data well? Justify your answer.

Yes, the residuals are small and random, and the linear regression line does seem to be a "good fit."

6. Compute the sum of the squares of the residuals for your data. Explain the process you went through to accomplish this.

Sum of the Squares of the Residuals = 446.468

Used the command,* SUM($L4^2$) *to calculate the value
***{could have used* SUM($_L RESID^2$) }**

DATA ANALYSIS

Entering Data:

1. STAT
2. Choose EDIT
3. Clear needed lists:
   a) to put the cursor on the list name
   b) CLEAR
   c) ENTER
4. Enter *x*'s in L1 and *y*'s in L2 (or other lists as appropriate)

Constructing a Scatter Plot:

1. STATPLOT (2nd, Y=)
2. PLOT 1 (or other plot as appropriate)
3. ENTER to turn on plot
4. , ENTER to choose a scatter plot
5. , 2 nd, 1 to put L1 in the Xlist (or other list as appropriate)
6. , 2 nd, 2 to put L2 in the Ylist (or other list as appropriate)
7. , ENTER to choose the box mark
8. ZOOM
9. ZOOMSTAT (Graph appears in calculator's window)

As required, press WINDOW to see or change your window or scale.

Finding the Best Fit Model:

1. Turn on Diagnostics
   a) 2nd, 0 (Catalog)
   b) DiagnosticsOn, Enter
2. STAT
3. to choose CALC
4. LINREG (ax + b) (or other regression model as appropriate)
5. Indicate the location of your data, and where to put the resulting model {i.e., LinReg (ax+b), L1, L2, Y1}:
   a) 2nd, 1 to indicate L1 as the location of the *x*'s (or other list as appropriate)
   b) , (comma)
   c) 2nd, 2 to indicate L2 as the location of the *y*'s (or other list as appropriate)
   d) , (comma)
   e) VARS
   f) to choose Y-VARS
   g) FUNCTION
   h) Y1 (or other function as appropriate)
6. ENTER
7. GRAPH to see the model graphed with the data.

Making Predictions:

Use the TABLE. (Put the cursor in the Y column to view exact value. May want to use TBLSET to start the table at the desired *x* value.)

OR

Use the home screen to substitute an *x* value into the regression equation. (To avoid rounding error, use the VARS-STATISTICS-EQ menu to retrieve regression results. For example, if performing LinReg(ax+b) and substituting in 2 for x, type "a*2+b" in the home screen.)

Analyzing the Residuals:

1. STAT
2. Go to L3 (or other empty list as appropriate)
3. to put the cursor on the list name
4. LIST (2^{nd}, STAT)
5. RESID
6. Construct a scatter plot of the *x* values versus residuals.

   NOTE: The best fit model will have the residuals scattered (not in some pattern) and the residuals are small in value.

   CAUTION: The residuals are automatically calculated after each regression and stored in list, RESID. The residuals are only based on the last calculated regression. Inserting a new set of data will not change the RESID list.

Analyzing the Sum of the Squares of the Residuals:

1. LIST (2^{nd}, STAT)
2. to MATH
3. SUM(
4. $L3^2$) (is accomplished by hitting the 2^{nd}, 3, and x^2 keys)

   NOTE: The regression model with the lowest sum of the squares of the residuals indicates the best fit model.

NAME ___________________________________
DATE ________________

WORKSHEET #2: PENDULUM LAB

Materials Needed:

- String (approx. 6 feet of string for each group—approx. 10-12 groups)
- Yard stick
- Heavy washer (to act as the "bob," or weight, at the end of the string)
- Stopwatch
- Protractor
- TI-83 (or TI-83 PLUS)

In this experiment, you will form a group of 3 (with 2 other classmates), and you will be investigating the properties of pendulums as you are collecting experimental data. The data that you are to collect is the amount of time it takes a pendulum to complete 5 entire periods. Your group will select 4 different lengths of string for the pendulum, and perform 3 time trials for each length chosen. Record your answers in the worksheet below. Compute the average time per period (average of the 3 trials divided by 5) at each of the 4 lengths, and record that answer.

PENDULUM LAB--RESULTS FOR YOUR <u>INDIVIDUAL</u> GROUP

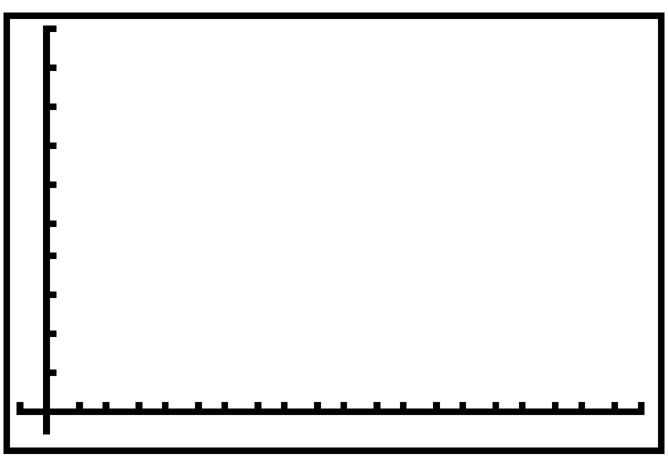
TRIAL #	LENGTH OF STRING (FEET)	AMT. OF TIME (SEC) FOR ALL 5 PERIODS	AVG. TIME (SEC) *PER PERIOD*
1		**1st:** **2nd:** **3rd:**	
2		**1st:** **2nd:** **3rd:**	
3		**1st:** **2nd:** **3rd:**	
4		**1st:** **2nd:** **3rd:**	

All of the groups in the class will then record their **average** results on the blackboard for each of the 4 lengths of string they chose. Each group will then select 10 data points of different lengths from the blackboard, and will record those 10 data points in the chart below. Analyze this data by performing the following instructions and answering the associated questions.

DATA FOR THE ENTIRE CLASS (CHOOSE ANY 10 POINTS)

DATA POINT	LENGTH OF STRING (FEET)	AVERAGE PERIOD (SEC)
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		

1. Under Stat Edit, enter your class' experimental data into lists L1 and L2 using your TI-83 graphing calculator. Sketch your scatter plot below.



2. The data points *may look like* they form a linear relationship, but before deciding that, your group needs to investigate the relationship between the time (T) and the length of the string used (L). In order to do that, choose 10 data points and complete the chart below which explores the experimental values for T/L, T^2/L, and T^3/L. (The purpose of this chart is to determine what regression should be pursued.)

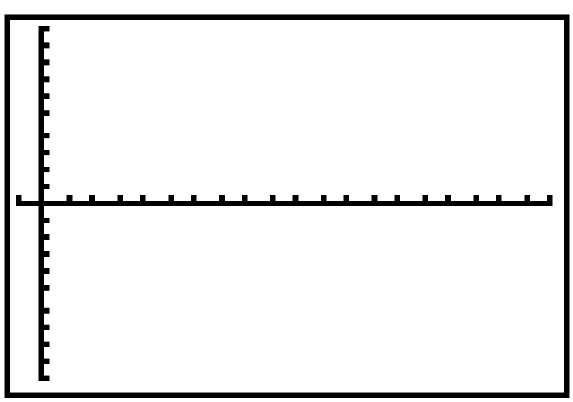
TIME(T)	LENGTH (L)	T/L	T^2/L	T^3/L

What do you notice about the last 3 columns above (relative to the range in values)? What is your conclusion about what type of regression test to use for this pendulum data?

3. Based on your answer to these last two questions, you need to determine the equation of the "best fit **curve**" using these 3 regression tests: power, quadratic, and exponential. Be sure to plot your residuals on the graph below each category. (Remember that when you have two or more sets of residuals, the smallest sum of the squares of the residuals represents the "best fit" curve).

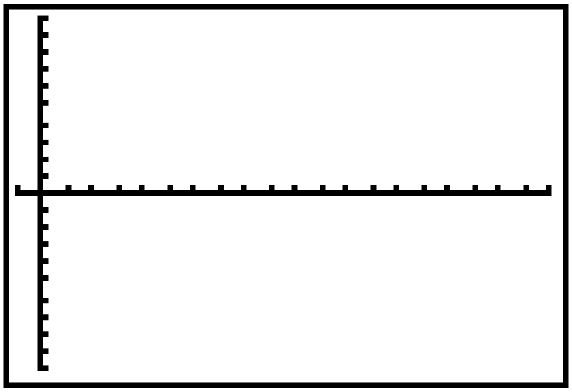
(a) POWER EQ: ______________________________________

r = __________ Sum of squared residuals: _________



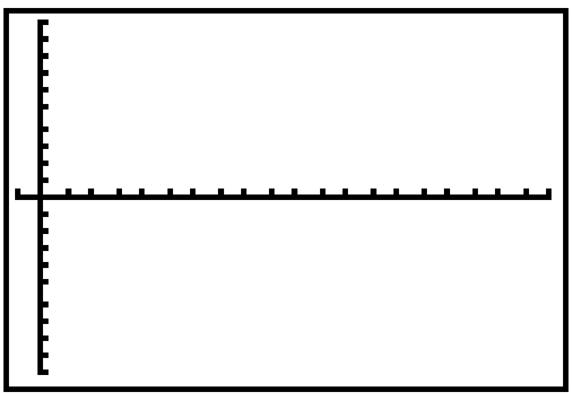
(b) QUADRATIC EQ: ________________________________

r = _________   Sum of squared residuals: _________

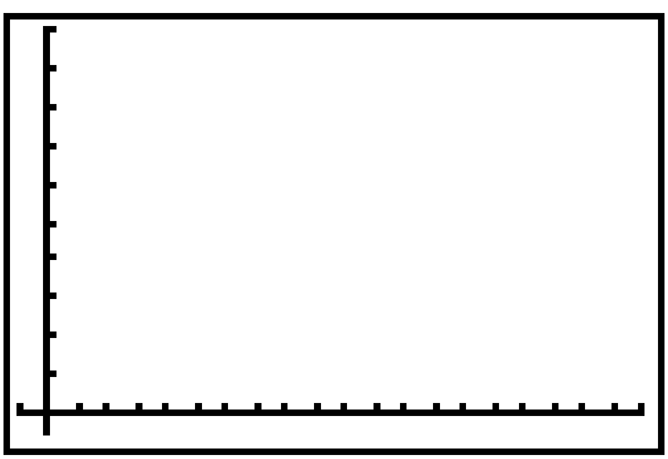


(c) EXPONENTIAL EQ: ______________________________

r = _________   Sum of squared residuals: _________

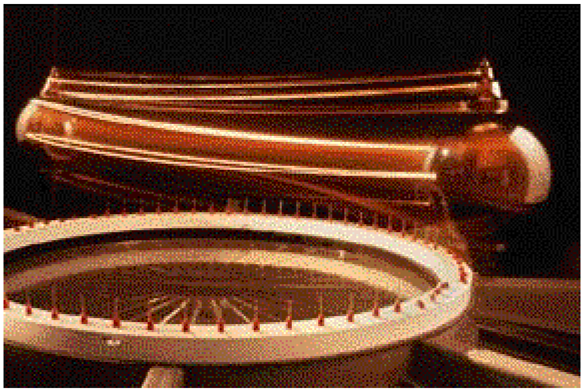


4. Construct a scatter plot of the original data points and the 3 regression equations. Zoom out three times to observe the effect of the regression equations over time. Then, sketch this view on the graph below.



5. Based on the correlation coefficients, residuals, sum of the residuals squared, and the graphs of the curve of best fit, which of the 3 equations is really the "best fit curve" and why?

6. The pendulum at the National Museum of American History is modeled after J. B. L. Foucault's 1851 pendulum. A 54-foot steel cable from the ceiling of the fourth floor suspends its 240-pound hollow bob. What do you expect the period of this pendulum to be?



ANSWER KEY

WORKSHEET #2: PENDULUM LAB

Materials Needed:

- String (approx. 6 feet of string for each group—approx. 10-12 groups)
- Yard stick
- Heavy washer (to act as the "bob," or weight, at the end of the string)
- Stopwatch
- Protractor
- TI-83 (or TI-83 PLUS)

In this experiment, you will form a group of 3 (with 2 other classmates), and you will be investigating the properties of pendulums as you are collecting experimental data. The data that you are to collect is the amount of time it takes a pendulum to complete 5 entire periods. Your group will select 4 different lengths of string for the pendulum, and perform 3 time trials for each length chosen. Record your answers in the worksheet below. Compute the average time per period (average of the 3 trials divided by 5) at each of the 4 lengths, and record that answer.

PENDULUM LAB--RESULTS FOR YOUR INDIVIDUAL GROUP

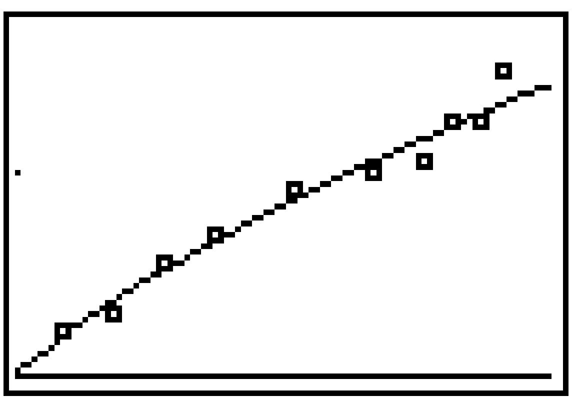
TRIAL #	LENGTH OF STRING (FEET)	AMT. OF TIME (SEC) FOR ALL 5 PERIODS	AVG. TIME (SEC) *PER PERIOD*
1	*4.33 feet*	*1st: 12 sec* *2nd: 12 sec* *3rd: 12 sec*	*2.4 sec*
2	*4.167 feet*	*1st: 11 sec* *2nd: 11 sec* *3rd: 11 sec*	*2.2 sec*
3	*3.5 feet*	*1st: 10 sec* *2nd: 10 sec* *3rd: 10 sec*	*2 sec*
4	*3 feet*	*1st: 9.5 sec* *2nd: 10 sec* *3rd: 9.5 sec*	*1.933 sec*

All of the groups in the class will then record their **average** results on the blackboard for each of the 4 lengths of string they chose. Each group will then select 10 data points of different lengths from the blackboard, and will record those 10 data points in the chart below. Analyze this data by performing the following instructions and answering the associated questions.

DATA FOR THE ENTIRE CLASS (CHOOSE ANY 10 POINTS)

DATA POINT	LENGTH OF STRING (FEET)	AVERAGE PERIOD (SEC)
1	4.33 ft.	2.4 sec
2	4.167	2.2
3	4	2.2
4	3.833	2.06
5	3.5	2
6	3	1.934
7	2.5	1.766
8	2.167	1.666
9	1.833	1.466
10	1.5	1.4

1. Under Stat Edit, enter your class' experimental data into lists L1 and L2 using your TI-83 PLUS graphing calculator, and sketch your scatter plot below.



2. The data points may look like they form a linear relationship, but before deciding that, your group needs to investigate the relationship between the square of the time (T) and the length of the string used (L). In order to do that, choose 10 data points and complete the chart below which explores the experimental values for T/L, T^2/L, and T^3/L.

TIME(T)	LENGTH (L)	T/L	T^2/L	T^3/L
2.4	**4.33**	**.5543**	**1.3303**	**3.1926**
2.2	**4.167**	**.5280**	**1.1615**	**2.5553**
2.2	**4**	**.5500**	**1.2100**	**2.6620**
2.06	**3.833**	**.5374**	**1.1071**	**2.2807**
2	**3.5**	**.5714**	**1.1429**	**2.2857**
1.934	**3**	**.6447**	**1.2468**	**2.4113**
1.766	**2.5**	**.7064**	**1.2475**	**2.2031**
1.666	**2.167**	**.7688**	**1.2808**	**2.1339**
1.466	**1.833**	**.7998**	**1.1725**	**1.7189**
1.4	**1.5**	**.9333**	**1.3067**	**1.8293**

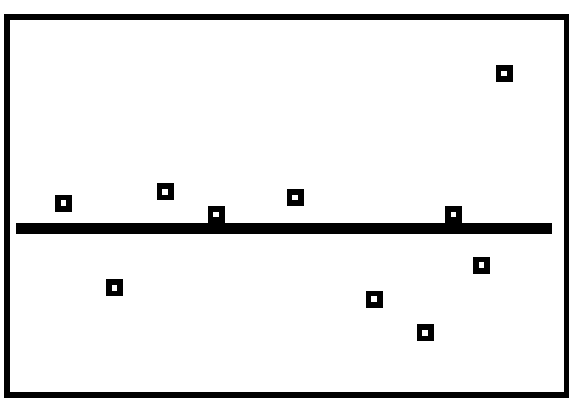
What do you notice about the last 3 columns above (relative to the range in values)? What is your conclusion about what type of regression test to use for this pendulum data?

The T/L and T^3/L columns show wide variation in their values, while the T^2/L column shows relatively the same numbers (within a small range). The conclusion is that the data is non-linear (i.e., quadratic, power, or exponential).

3. Based on your answer to these last two questions, you need to determine the equation of the "best fit **curve**" using these 3 regression tests: power, quadratic, and exponential. Be sure to plot your residuals on the graph below each category.

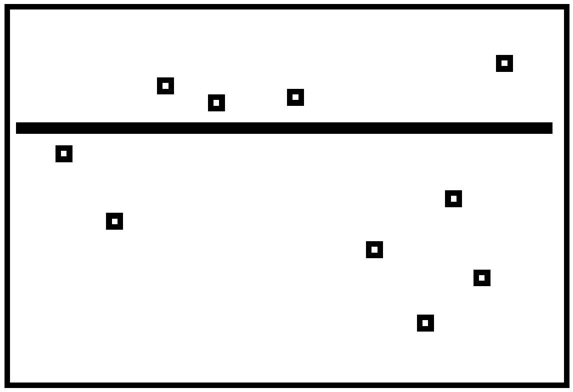
(a) POWER EQ: $y = (1.13653)\,x^{0.47271}$

r = ***.9872*** Sum of squared residuals: ***0.0320***



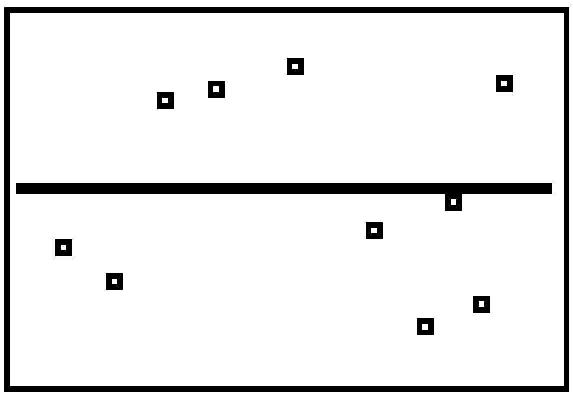
b) QUADRATIC: ***y = (-.002777) x^2 + (.331135) x +0.917360***

r = ***.9846*** Sum of squared residuals: ***0.0299***

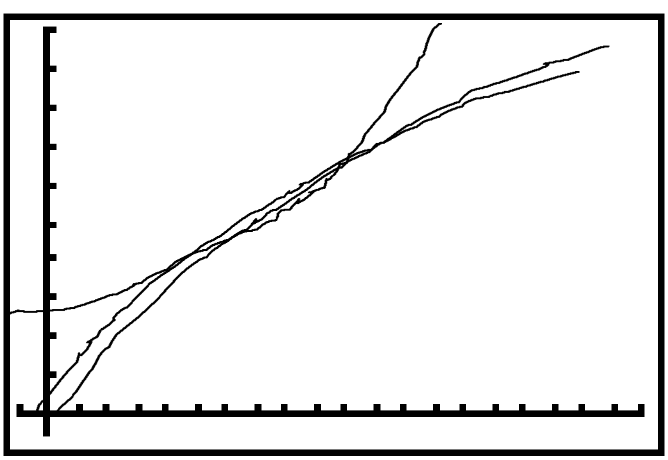


c) EXPONENTIAL: ***y = 1.11088*1.18659^X***

r = ***.9835*** Sum of squared residuals: ***0.0335***



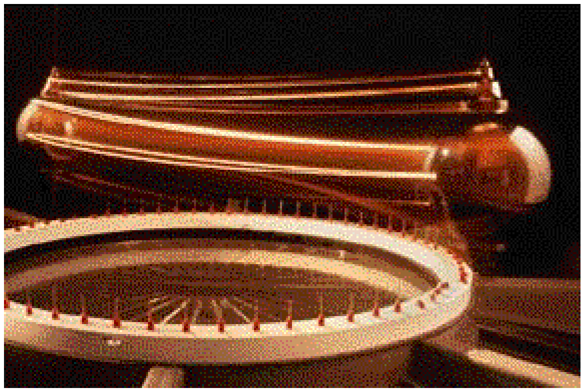
4. Construct a scatter plot of the original data points and the 3 regression equations. Zoom out three times to observe the effect of the regression equations over time. Then sketch this view on the graph below.



5. Based on the correlation coefficients, residuals, sum of the residuals squared, and the graph of the curves of best fit, which of the 3 equations is really the "best fit curve" and why?

   The power regression appears to be the best. It has the highest correlation coefficient, the residuals are small relative to the data, the residual plot is random in pattern, the sum of the residuals squared is small, and the graph reflects what will happen as length increases. Note, the quadratic model shows a decrease in time as length increases.

6. The pendulum at the National Museum of American History is modeled after J. B. L. Foucault's 1851 pendulum. A 54-foot steel cable from the ceiling of the fourth floor suspends its 240-pound hollow bob. What do you expect the period of this pendulum to be?



   $$y = (1.3653)\ (54)^{0.47271}$$

   $$y = 7.4903 \text{ sec}$$

ASSESSMENT

Teacher's Guide

Introduction

The assessment should be given at the end of this unit.

Objectives Covered

Students will:

- construct a scatter plot.
- calculate line of best fit using the TI-83 graphing calculator.
- perform predictions using the equation for the line of best fit.
- analyze regression using correlation coefficient, residual plot, sum of the squares of residuals, and the scatter plot with the line of best fit.

Tools/Materials Needed for Assessment

- TI-83 Graphing Calculator
- Line of Best Fit Assessment Sheet

Administering the Assessment

Students should have one 45-minute class period to complete the assessment. It will require calculations and graphing using the TI-83. The results and any required explanations will be documented on the Assessment Sheets. The answer key and rubric are included.

ASSESSMENT

Problem 1

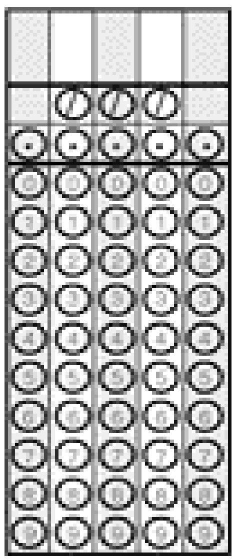
The gearing your grandfather clock requires the pendulum swing to have a period of two seconds. The theoretical relationship between the pendulum period and length is

$$t = 2\eth\sqrt{\frac{l}{g}}$$

where:

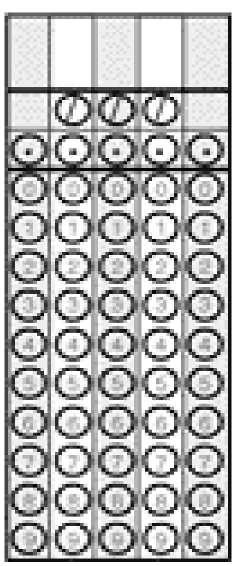
t = Period of the pendulum swing in seconds
l = Length of the pendulum in feet
g = 32 feet / sec^2.

How long is the grandfather clock's pendulum? Grid in your answer in the grid below.



Problem 2

Using the same theoretical equation above, determine the period of the pendulum, if the length of the pendulum in your parents' mantel clock is 3 inches. Grid in your answer in the grid below.



ASSESSMENT -- KEY

Problem 1

The gearing your grandfather clock requires the pendulum swing to have a period of two seconds. The theoretical relationship between the pendulum period and length is

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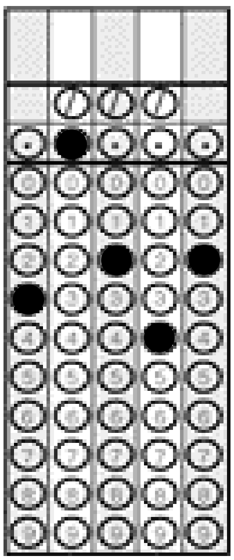


$$t = 2\eth\sqrt{\frac{l}{g}}$$

$$\frac{t}{2\eth} = \sqrt{\frac{l}{g}}$$

$$\frac{t^2}{4\eth^2} = \frac{l}{g}$$

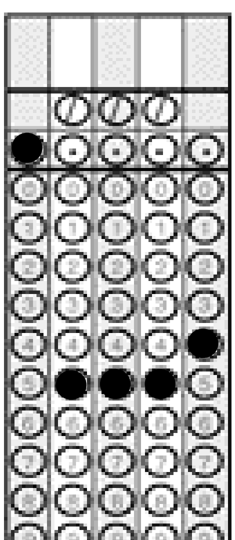
$$\frac{gt^2}{2\eth^2} = l$$

$$\frac{(32)(2)^2}{4\eth^2} = l$$

$$3.242\ ft = l$$

Problem 2

Using the same theoretical equation above, determine the period of the pendulum, if the length of the pendulum in your parents' mantel clock is 3 inches.



$$t = 2\eth\sqrt{\frac{l}{g}}$$

$$t = 2\eth\sqrt{\frac{(3/12)}{32}}$$

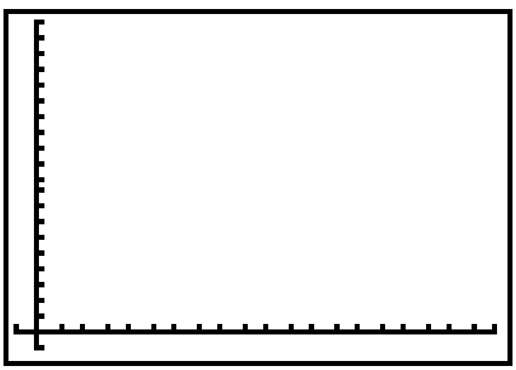
$$t = .5554\text{sec}$$

<u>Problem 3--Extended Constructed Response (to be graded using the 4-point HSA rubric)</u>

The table below shows the total number of Electronic Funds Transfers (EFT) per year from 1985 to 1996. (NOTE: EFTs include Automated Teller Machine (ATM) transactions as well as transactions at Point of Sale (POS) terminals.)

YEAR	NUMBER OF TRANSACTIONS (MILLIONS)
1985	3579
1990	5942
1991	6642
1992	7537
1993	8135
1994	8958
1995	10464
1996	11830

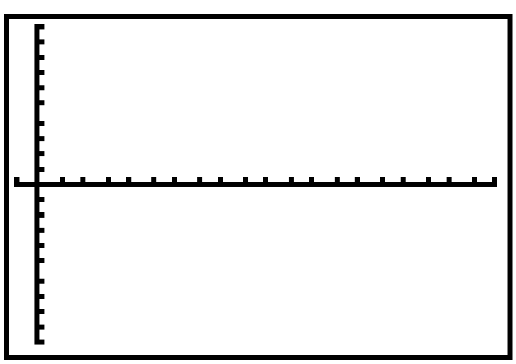
1. Construct a scatter plot of the data above, and sketch it below. **When entering the years, let 1985 equal 1. Adjust the other years, accordingly**.



2. Perform the following regressions, showing your regression equations, r values, sums of the squared residuals, and scatter plots.

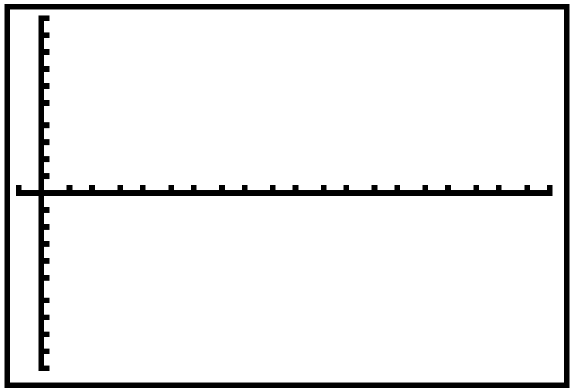
   a) QUADRATIC--Regression Eq: ___________________________________

   r =_____________ sum of squares of residuals = ____________



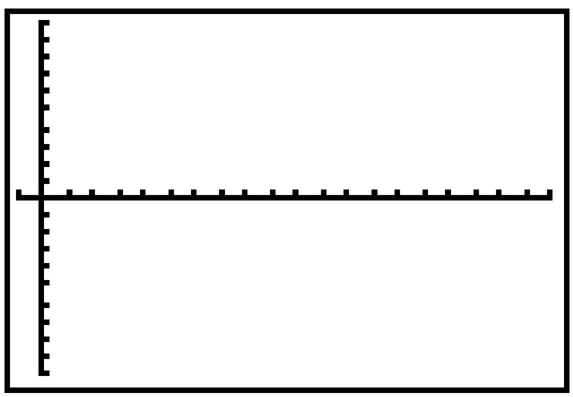
b) POWER--Regression Eq: ____

r = ____________ sum of squares of residuals = ____________



c) EXPONENTIAL--Regression Eq: _________________________________

r = ____________ sum of squares of residuals = ____________



3. Which of the above regression equations is the "best fit curve?" Justify your answer using mathematics.

4. Based on your selected model, what is the predicted number of EFT transactions for the year 2000? Explain how you arrived at your answer. Use words, symbols, or both in your response.

Assessment

Scoring Guide

Rubric for the Extended Constructed Response

4	**Student demonstrates a knowledgeable understanding of finding and using a non-linear function to model data.** Graph is labeled, scaled, and accurate Accurately identifies proper non-linear model Accurately uses model to find domain and range values Correctly identifies the dependent and independent variables Answers include units Correctly identifies and explains the prediction
3	**Student demonstrates an understanding of finding and using a non-linear function to model data.** Graph is labeled or scaled, and accurate Uses non-linear model to find domain and range values with minor mistakes Correctly identifies the dependent and independent variables Most answers include units Correctly identifies prediction, explanation is incomplete/confusing
2	**Student demonstrates some understanding of finding and using a non-linear function to model data.** Graph is labeled or scaled, and has minor errors Uses non-linear model to find domain and range values with mistakes Correctly identifies the dependent and independent variables Some answers include units Errors in identifying the prediction and explanation is not clear
1	**Student demonstrates little understanding of finding and using a non-linear function to model data.** Makes an attempt to graph Knows to use non-linear model to find domain and range values Makes attempt to predict
0	**Student makes no attempt to solve the problem.**

ANSWER KEY

Problem 3--Extended Constructed Response (to be graded using the 4-point HSA rubric)

The table below shows the total number of Electronic Funds Transfers (EFT) per year from 1985 to 1996. (NOTE: EFTs include Automated Teller Machine (ATM) transactions as well as transactions at Point of Sale (POS) terminals.)

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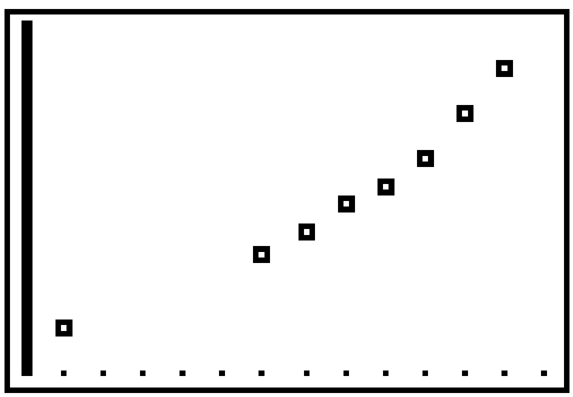
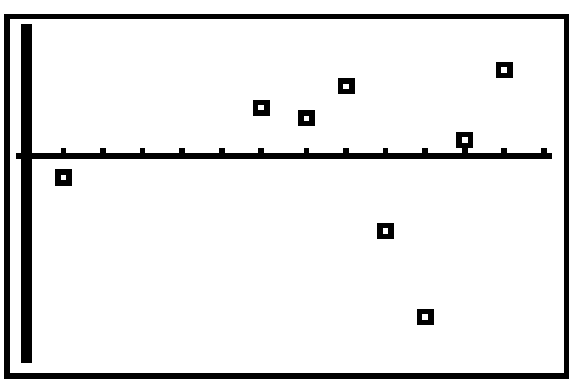


2. Perform the following regressions, showing your regression equations, r values, sums of the squared residuals, and scatter plots.

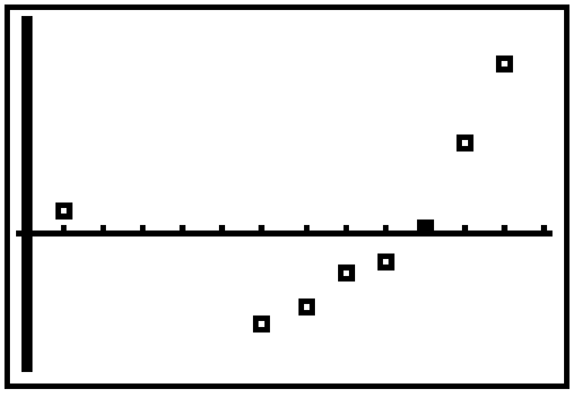
   a) QUADRATIC--Regression Eq: **$y = (47.921)\ x^2 + (105.406)\ x + 3472.5$**

   r =**0.9949** sum of squares of residuals = **241715.62**



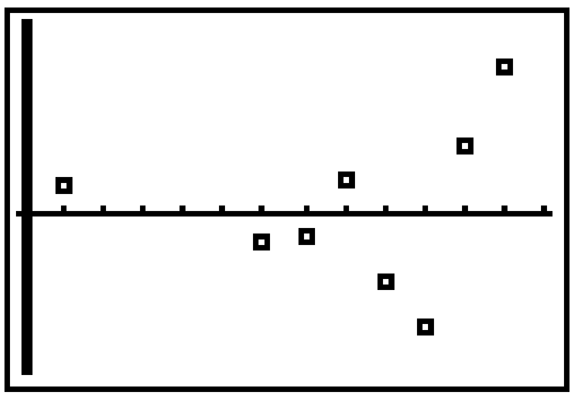
b) POWER--Regression Eq: **$y = (3267.28)\ x^{.4346}$**

r = **0.93434** sum of squares of residuals = **9153362.488**



c) EXPONENTIAL--Regression Eq: **$y = (3158.909)\ (1.1134)^{x}$**

r = **0.99828** sum of squares of residuals = **292962.3016**



3. Which of the above regression equations is the "best fit curve?" Justify your answer using mathematics.

   The quadratic regression equation is the "best fit curve"--its sum of squared residuals is significantly lower than the exponential regression's sum

4. Based on your selected model, what is the predicted number of EFT transactions for the year 2000? Explain how you arrived at your answer. Use words, symbols, or both in your explanation.

   The predicted number of EFT transactions is 17,427 million--you just substitute 16 in for *x* in the quadratic regression equation (if 1985 is year 1, then 2000 is year 16).